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User Equilibrium in a Disrupted Network with Real-Time Information and Heterogeneous Risk Attitude

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USER EQUILIBRIUM IN A DISRUPTED NETWORK WITH REAL-TIME INFORMATION AND HETEROGENEOUS RISK ATTITUDE

A Thesis Presented

By

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Approved as to style and content by:

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Song Gao, Chair

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John Collura, Member

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Richard Palmer, Department Head
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DEDICATION

I would like to dedicate this thesis to my loving and supportive family for cheering me on through this intense yet very rewarding experience. Special dedication goes to my parents, Sharon and George, whose advice has always succeeded in pushing me in the right direction, and to my sister, Jessica, who helped give me a perfect dose of light-hearted encouragement to make my thesis a fun experience with lots of laughs.

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ABSTRACT

USER EQUILIBRIUM IN A DISRUPTED NETWORK WITH

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MAY 2012

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The traffic network is subject to random disruptions, such as incidents, bad weather, or other drivers' random behavior. A traveler's route choice behavior in such a network is thus affected by the probabilities of such disruptions, his/her attitude towards risk, and real-time information on revealed traffic conditions that could potentially reduce the level of uncertainty due to the disruptions. As the road network's performance is determined collectively by all travelers' choices, it is also affected by these factors. This thesis features the development of a multi-class user equilibrium model based on heterogeneous risk attitude distributions and a user equilibrium model based on various disruption probabilities and information penetration rates that can be used to perform sensitivity analyses for a traffic network. The method of successive average (MSA) is used to solve for the equilibrium conditions. Laboratory experimental data are used to calibrate the risk attitude model. A sample sensitivity analysis is performed to show the disruption and information penetration effects on network performance. Initial calibrations show promising results for route flow predictions in a congested network with respect to heterogeneous attitude. With respect to disruption probability and information access, having too

high information penetration will not improve the network's performance, while having a small disruption probability can improve traffic conditions in the network.

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CHAPTER 1

INTRODUCTION

People make route choices every day in any given road network for various reasons (minimize travel time, avoid congestion, etc.). Information about the traffic conditions in the network, e.g., that provided by an advanced traveler information system (ATIS), could potentially enable travelers to make better route choices that can help them satisfy their goals more effectively. However, what most do not realize is that their route choice and the route choices of others in the network have an impact on the overall performance of the network. The well-known Braess Paradox states that adding capacity to a network with the intention to alleviate poor traffic conditions can actually decrease the network's efficiency as travelers will consider minimizing their personal travel time without considering the effects it has on the network. Likewise, adding more information about traffic conditions in the network could create similar effects.

Additionally, another important factor that could affect the performance of a network is travelers' risk attitude when making a route choice. Travelers' sensitivity to traffic conditions on a network determines whether or not they will take a chance on minimizing their travel time at the possible cost of greater personal delay, avoid the situation altogether, or remain indifferent to the matter. These outlooks affect every traveler's route choice, and when applied to the entire population, have an effect on the efficiency of the network.

1.1 Research Objectives

The thesis presents the development of two models that predict user equilibrium conditions for a given network based on heterogeneous risk attitude and information access, respectively. Heterogeneous risk attitude incorporates multiple risk attitudes (risk-seeking, risk-averse, or risk-neutral) across all users in a route choice model, unlike homogeneous risk attitude, which only incorporates one. The risk attitude model is calibrated using experimental data from the route choice study performed in Lu et al. (2012). The traveler information model has theoretical applications on the effects of information access and disruption probability on the performance of the road network.

1.2 Contributions

There is a gap in the development of traffic equilibrium models in a stochastic network. Travelers inherently differ in terms of their risk attitude and access to real-time information. There are some equilibrium models that incorporate risk attitude (homogeneous or heterogeneous) and those that incorporate information access and disruption probability (Mirchandani and Soroush, 1987; Lo and Tung, 2003; Gao, 2005; Ukkusuri et al., 2006). However, there has been no research on the development and calibration of an equilibrium model that includes both heterogeneous risk attitudes and information access. The research proposed contributes to the start of the art by tackling two important sub-problems whose solutions can be later combined to build such a model:

1. Development and calibration of a user equilibrium model with heterogeneous risk attitude based on the expected utility theory.
2. Development of a model showing the combined effect of information penetration and disruption probability on individual route choice and network performance.

1.3 Literature Review

For the purpose of this research, it is important to research important studies concerning modeling user equilibrium, heterogeneous risk attitude, and route choice models based on information access.

1.3.1 User Equilibrium Models

In any general network, travelers want to choose the route that best suits their needs (e.g. minimize travel time, avoid congestion, etc.); however, they do not always choose the best route. Also, while a route may not always be the best choice for a traveler, it does have a chance to be. Probabilistic choice theory assumes that both travelers want to choose the best route and each route has a chance of being the best. Application of probabilistic choice theory can then be used in the development of traffic assignment models that can create stochastic user equilibrium (SUE) conditions (Daganzo and Sheffi, 1977). Under SUE, flows are assigned to routes based on the probability that each route will be chosen. Logit models (Dial, 1971; Fisk, 1980; Bell, 1995; Maher, 1998) assume the error terms of each route choice are independently and identically distributed Gumbel variables, allowing for a closed form probability (Ben-Akiva and Lerman, 1985). The distribution assumption can be troublesome for route choice modeling, however, as random errors of different overlapping routes can have high correlations and different variances (Sheffi, 1985). Probit models assume travel time disruptions follow multivariate normal distributions, allowing for flexible variance and covariance relationships. This makes the probit model easily applicable for route choice predictions. The model lacks a closed form probability like logit models and can have limited large-scale applications due to running costs (Nie, 2011).

These models in their original forms, however, do not take into account disruptions to the network explicitly and the underlying travel times are deterministic.

1.3.2 Risk Attitude Models

The number of studies done on modeling risk attitude is extensive, yet the evaluation methods for risk attitude studies can result in different classifications of risk attitude (Slovic 1964; MacCrimmon and Wehrong 1990). A reason for this is because risk attitude may not be detectable by just looking at people's choices. Other situational factors can cause different classifications (Schoemaker 1990). Weber et al. (1997) surmise from their study that three methods that can help measure risk attitude in various ways: expected utility, relative risk attitude, and perceived risk attitude.

Utility is the relative benefit or usefulness an object has for an individual. When applied to a set of risky alternatives, such as a gamble, this benefit is known as expected utility. When analyzing a set of choices, it is hypothesized that an individual will pick an alternative that maximizes the expected utility. The attitude of an individual can be determined by the choices made during each gamble proposed and formulating a utility formula for that person. Pratt (1964) and Arrow (1971) helped define the characterization of risk attitudes through use of utility formulas. There are two assumptions associated with expected utility theory: 1) risk preferences can be described by a utility function known only by the modeler and 2) attitudes towards risk can be rationalized by the expected utility function. Expected utility can be very useful in identifying and describing people's choice patterns over a specified period, but quantifying the person's risk attitude is relatively insignificant (Weber et al. 1997).

One theory that can illustrate the disadvantages in expected utility theory is called cumulative prospect theory (CPT), which states that individuals have a biased perception of the probabilities in a given choice. According to CPT, people tend to over-estimate small probabilities and under-estimate large probabilities (Kahnneman and Tversky 1979).

Relative risk shows that the differences in risk attitude with respect to expected utility may be a result of the differences in marginal values. Risk preferences may remain unchanged with this approach, resulting in a more stable perception of risk. Using relative risk can measure people's attitudes towards uncertain outcomes rather than certain outcomes (Dyer and Sarin 1982).

Perceived risk attitude is the assumption that decision makers are attracted or repelled by alternative choices that they feel are riskier than choices they feel are less risky (Weber and Bottom 1989). Perceived risk attitude has significantly stronger cross-situational stability than both expected utility and relative risk. This method is preferable for measuring peoples' tendency to choose between risky and safe choices (Weber and Milliman 1997).

While there are numerous studies on various models to determine peoples' risk attitude, there is not much research detailing the calibration of various risk attitude distributions to a find a general trend in traveler behavior. There are, however, many experiments with results that show a possible pattern in traveler risk attitude. Weber and Bottom (1989) determined peoples' risk attitudes by examining their choices on theoretical lotteries with various probabilities. Using CPT, they found that 76% of all participants were either risk-averse or risk-neutral. Another study, de Palma and Picard

(2005), concluded that 66% of participants were risk-averse or risk-neutral and 33% were risk-seeking. Weber and Milliman (1997) used expected utility theory to characterize traveler risk attitude in one of their experiments. The experiment required participants to choose various commuting times for a train to determine their utility functions for risk assessment. By observing the shape of each participant's utility function, it was found that a majority of participants were risk-averse or risk-neutral when the commuting times were slower than or equal to the average. When commuting times were faster or equal to the average, however, a majority of participants were found to be either risk-seeking risk-neutral. The results of these studies and others (Bruinsma et al. 1999; Lam and Small 2001) show a strong indication that people tend to be risk-averse.

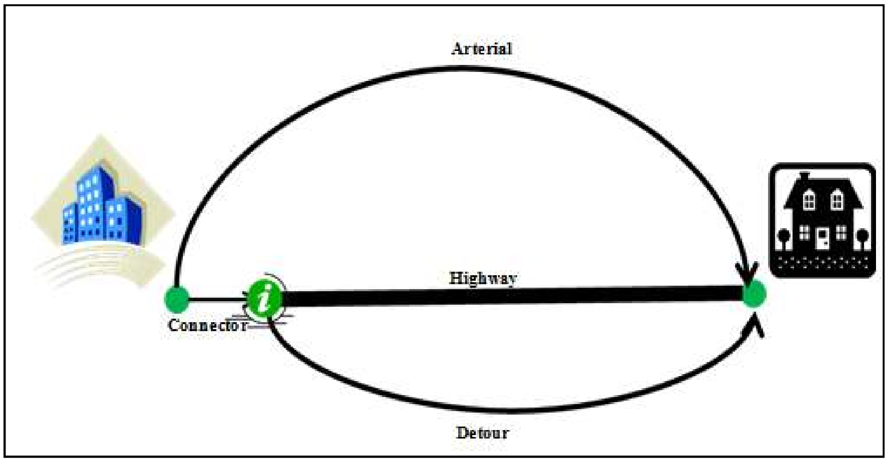
1.3.3 Real-Time Traveler Information Models

Pre-trip and en-route information can allow travelers to plan and adapt their trip to effectively meet their needs. Travelers who have access to traffic information are more likely to follow the provided pre-trip and en-route traffic information (Abdel-Aty and Abdalla, 2006). En-route real-time travel information allows travelers to make route choices at decision nodes based on current conditions to avoid delay (McQueen et al., 2002). With regards to en-route short-term choices, providing qualitative information is more beneficial for travelers than quantitative information (Abdel-Aty and Abdalla, 2006). Ukkusrui et al. (2006) concluded that any change in user behavior due to real-time information must be account for in traffic assignment models.

Models that incorporate information access can give further insight into traveler route choice behavior. Gao et al. (2008) describe two route choice models that incorporate real-time information: adaptive path models and strategic route choice models. Adaptive path models assume route choices are a series of path choices at every decision node. This can account for diversion from the initial path but does not plan ahead for upcoming information. Strategic route choice models are based on a rule that maps stochastic network conditions to routing decisions. This model assumes travelers have expectations for en-route travel information. It also assumes that travelers are proactive when planning routes.

CHAPTER 2

EXPERIMENT DESIGN



Experiment Road Network

The development and calibration of the user equilibrium models were based on data obtained from the route choice experiment featured in Lu et al. (2012). The experiment was composed of eight sessions, each involving the participation of 16 individuals. During each session, participants were instructed to make route choices from "work" to "home" in the network, shown in Figure 1, on a day-to-day basis. They were shown the free-flow travel times of each route before the start of the experiment and were told that the highway has a disruption probability of 0.25. While not informed of the duration of the experiment to prevent bias, participants made route choices for 120 days. After every participant made a route choice for a given day, they were shown the travel time for their chosen route.

The experiment was split into two scenarios, each composed of four sessions. The first scenario, known as the incident case, had participants make route choices without being informed if the highway was experiencing an incident that day that would greatly increase the travel time on that path. The second scenario, known as the information case, provided an information node for travelers who chose the connector. Once at the information node, participants were told whether or not the highway was experiencing some form of disruption that day. The participants were then able to make an informed decision based on the traffic conditions on the highway.

CHAPTER 3

MODELING USER EQUILIBRIUM WITH HETEROGENEOUS RISK ATTITUDE

The following section details the development of the user equilibrium model that incorporates heterogeneous risk attitude in its predictions. This chapter will detail the conditions for user equilibrium with respect to heterogeneous risk attitude, the solution algorithm used to derive the user equilibrium, and the assumptions used to calibrate the model.

3.1 User Equilibrium Conditions with Heterogeneous Risk Attitude

The conventional user equilibrium condition in a static and deterministic network is generalized to the stochastic network with heterogeneous user risk attitude (with no real-time information) in this thesis as follows: at user equilibrium, all used paths have the same and minimum expected disutility for each origin-destination pair and risk attitude class. For the network in Figure 1, user equilibrium for the incident case is met when the expected disutilities for all three paths (the arterial, highway, and detour) are equal and minimized.

It was previously stated that when an individual faces a set of choices, he/she will pick an alternative that maximizes the expected utility. However, in the context of route choices in a road network, travelers are assumed to minimize their expected disutility, or the relative cost a choice has for an individual. The reason expected disutility is used instead of expected utility is that regardless of choice, a route is associated with a cost (e.g. travel time).

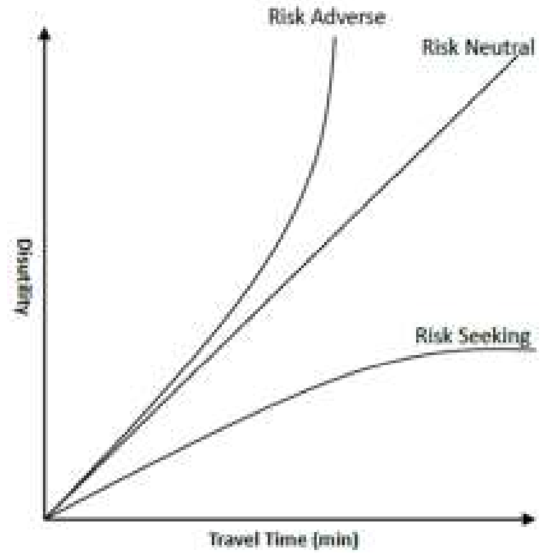


Figure 1: Risk Attitudes with Respect to Expected Disutility

The curvature of the disutility function of route travel times will characterize the traveler's risk attitude as shown in Figure 2. The expected disutility for a traveler on a chosen route is calculated by the following equation:

$$EDU = TT^c$$

$TT = Route\ Travel\ Time$ (minutes)

$c = Risk\ Attitude\ Parameter$

The c-value in the equation helps denote a traveler's risk attitude. Risk neutrality can be represented with disutility as the straight line shown in Figure 2, and the marginal disutility is constant. Under risk neutrality, a traveler's expected disutility for a route is equal to that route's travel time. Therefore, people who are risk neutral will have a c-value of 1.0. Travelers exhibiting a risk-averse attitude can be represented by the convex function, where the marginal disutility is increasing. As the marginal disutility is increasing, risk-averse travelers will have c-values greater than 1.0. The concave function best represents risk-seeking behavior, as the marginal disutility is decreasing. Risk seeking individuals will have c-values that are greater than zero but less than 1.0. The c-value is a parameter that must be calibrated from data.

Note that user equilibrium is usually used to describe a steady state of a traffic network. In a network subject to random capacity reduction, how to define a "steady state" is by itself a research question. In this thesis, the mean travel time (or disutility) taken over a large number of days is used to describe the state of the network, as by definition the travel time is a random variable distributed over days, and one cannot expect a fixed travel time from day to day.

3.2 Solution Algorithm

The solution algorithm used to derive equilibrium route choices (traffic flows) in the network is based on the method of successive averages (MSA). MSA is an iterative process that will heuristically solve for equilibrium conditions (Sheffi and Powell 1982). The algorithm will run multiple iterations and distribute flows to an optimal path with the smallest expected disutility for each risk group.

The first iteration for a given risk group begins by looking at the free flow travel time of every path in the network. The path with the lowest free flow travel time will then have the entire demand of the network. For the next iteration, the expected disutility for each path will be calculated. The path with the lowest expected disutility will then become the new optimal route choice. Flows from all paths are then re-allocated by the following algorithm.

1. **For All Paths:**

$$f_{ij}^{n} = \left(\frac{n-1}{n}\right) * f_{ij}^{n-1}$$

$f_{ij}^{n} = flow\ for\ route\ choice\ i\ at\ current\ iteration\ n\ for\ risk\ group\ j$

$f_{ij}^{n-1} = flow\ for\ route\ choice\ i\ during\ the\ previous\ iteration$

2. **Aggregate Re-allocated Flow:**

$$r_j^n = \sum_{i=1}^{p} \frac{f_{ij}^{n-1}}{n} = \frac{Q}{n}$$

$r_j^n = flow\ being\ rellocated\ to\ new\ optimal\ route\ choice\ for\ risk\ group\ j$

$p = total\ number\ of\ route\ choices$

$Q = total\ demand$

3. **Add Aggregate Flow to Optimal Routing Policy:**

$$f_{i*}^{n} = f_i^n + \sum_{j=1}^{k} r_j^n$$

$f_{i*}^{n} = flow\ on\ new\ optimal\ route\ choice$

$k = total\ number\ of\ risk\ groups$

Once all the flows have been re-allocated to the optimal route choice for a given risk group, the expected disutility for each route choice will be re-calculated. The difference between the current and previous iterations' expected disutilities for each route choice will be calculated. If the absolute value of this difference is less than or equal to the desired limit across all paths for each risk group, then the MSA analysis will stop and equilibrium conditions are met. Otherwise, the iteration process will be carried through until the limit condition is met.

3.3 Model Calibration

Two different types of calibrations were performed to develop risk attitude parameters and distributions. The first is a disaggregate analysis to derive a risk parameter for each participant based on the individual level route choices over the duration of the experiment. The second calibration is an aggregate analysis that calibrates the risk parameter distribution among all the participants of a given session based on aggregate path flows averaged over the steady period of the experiment. The disaggregate analysis is used to provide support for the risk parameter distribution assumptions in the aggregate analysis. Due to a limited number of combinations of parameter values, the disaggregate analysis is not yet complete but still able to provide results.

3.3.1 Disaggregate Analysis

The disaggregate calibration analysis attempts to characterize an experiment session's risk attitude distribution by characterizing each participant's route choice behavior and assigning a c-value based on their daily route choices. This is done by assessing what c-value and additional parameters produce the most accurate route choice predictions for each individual. The individual modeling is done by analyzing each day's route choice, calculating the expected disutility for each possible route choice for each day, calculating the probability of an individual choosing a specific route using a logit model, and calculating the likelihood the predicted route choices are correct based on the given parameters.

The input file contains the route choice information on a participant of an experiment session, including the participant's route choice, the route's travel time, and whether or not there was an incident on the highway for each day of the experiment. All 64 participants in the incident case were analyzed through this method.

When a participant's file is read into the program, they are assigned with three parameters that will vary throughout the calibration: a c-value, an arterial bias (α), and a probabilistic scale (λ). The c-value will help determine how risk-seeking or risk-averse an individual is. The arterial bias will capture any bias for the safe arterial not accounted for by the expected utilities, for example, to offset any complications caused by participants selecting the arterial just because less clicks were require to finish that day's route choice selection. Finally, the logit model scale is used to describe the sensitivity of the choice to the difference in expected disutilities.

Once all the parameters are assigned to an individual, the first step in the calibration algorithm is to analyze their route choices for each day of the experiment. For each day of the experiment, the model determines whether or not the arterial, detour, or the highway was chosen that day. If the highway was chosen on that day, the model also notes whether or not there was an incident on the highway that day. While noting each day's route choice, the model will also record the total number of times a route has been chosen by an individual as the model analyzes the remaining days of the experiment. The model will also record the total number of times the highway was chosen when there was an incident and when there was no incident.

After the day's route choice was recorded, each route's average expected disutility was calculated for that day. Calculating the expected disutility for the arterial and the detour for each day use the same procedure while calculating the expected disutility for the highway is more complex. If the arterial or the detour was chosen, then the expected disutility is calculated normally using the chosen route's travel time. If the route has been chosen on previous days, then its expected disutility is average with its previous disutilities to create a new average expected disutility for that day. If a route has not been chosen at all, then its expected disutility is calculated using its free-flow travel time. This is under the assumption that participants remember being shown each route's free-flow travel time at the beginning of the session. The route's expected disutility based on its free-flow travel time will serve as its average expected disutility until it has been chosen by the individual. If an individual chooses a previously unchosen route for the first time, then its expected disutility will become the new average expected disutility, as the individual has more accurate information on the route than their initial assumption.

While the process for analyzing the highway's average expected disutility is similar to the previous method used for the arterial and the detour, the incident probability for the network creates different conditions for calculating the expected disutility. As the highway is stochastic, there are four conditions for calculating the expected disutility for the highway, depending on the number of times the highway was chosen and whether or not an incident occurred on that day. If the highway has not been chosen, then the expected disutility is calculated using the following formula:

$$EDU_t^{high} = (1 - DP) * \left(TT_{free}\right)^c + DP * \left(TT_{free*}\right)^c$$

$$EDU_t^{high} = average\ highway\ expected\ disutility\ on\ day\ t$$

$$DP = disruption\ probability$$

$$TT_{free} = free - flow\ travel\ time\ with\ no\ incident$$

$$TT_{free*} = free - flow\ travel\ time\ with\ incident$$

The free-flow travel time (20 minutes) when there was no incident was provided for participants at the beginning of the experiment, but there was no description to describe how large the free-flow travel time would increase if there was an incident. It is assumed that participants could think of some unreasonably long travel time to visualize the highway during an incident, so using an incident free-flow travel time of 120 minutes was chosen for analysis. The expected disutility calculated with these free-flow travel times is used as the average expected disutility until the highway is chosen.

Once the highway is chosen, the expected disutility is calculated under three possible conditions. One condition is if the highway has only been chosen when there has not been an incident, where the expected disutility equation is:

$$EDU_t^{high} = (1 - DP) * \left(\frac{\sum_{j=1}^{k} TT_j^{norm}}{k_t}\right)^c + DP * \left(TT_{free*}\right)^c$$

$$TT_j^{norm} = normal\ travel\ time\ on\ j^{th}\ time\ it\ was\ chosen$$

$$k_t = total\ number\ of\ times\ highway\ was\ selected\ without\ incident\ by\ day\ t$$

The next condition is if the highway has only been chosen when there has been an incident. Its equation is given as:

$$EDU_t^{high} = (1 - DP) * (TT_{free})^c + DP * \left(\frac{\sum_{m=1}^{k^*} TT_m^{inc}}{k_t^*}\right)^c$$

$TT_m^{inc} = incident\ travel\ time\ on\ m^{th}\ time\ it\ was\ chosen$

$k_t^* = total\ number\ of\ times\ highway\ was\ selected\ with\ incident\ by\ day\ t$

Finally, the last expected disutility equation for the highway applies when the highway has been chosen during an incident and no incident at least once, calculated with the following equation:

$$EDU_t^{high} = (1 - DP) * \left(\frac{\sum_{j=1}^{k} TT_j^{norm}}{k_t}\right)^c + DP * \left(\frac{\sum_{m=1}^{k^*} TT_m^{inc}}{k_t^*}\right)^c$$

The analysis of a participant's route choice and the calculation of its respective expected disutility continue throughout the duration of the 120-day experiment, but the route choice prediction process on the model starts on the 31st set of calculations. This represents the end of the exploration period participants experience during the first 30 days of the experiment. The next 90 route choice analyses for the individual reflect the idea that individuals are making route choices that reflect their risk attitude.

The first step in the prediction process is to determine the probability that an individual will choose a specific route. A logit model is used to help determine the probability a route will be chosen each day. The probabilities are based on the previous day's average expected disutility for each route. The arterial bias is added to the average expected disutility to determine if the individual will be more preferential to the arterial than the other routes if everything else is equal. The average expected disutility for each route is multiplied by the scale and negated as the expected disutility is an associated cost and supposed to be minimized. The logit model equations for route choice probabilities are shown below:

$$P_t^{art} = \frac{e^{-\lambda(EDU_{t-1}^{art}+\alpha)}}{e^{-\lambda(EDU_{t-1}^{art}+\alpha)} + e^{-\lambda(EDU_{t-1}^{high})} + e^{-\lambda(EDU_{t-1}^{det})}}$$

$$P_t^{high} = \frac{e^{-\lambda(EDU_{t-1}^{high})}}{e^{-\lambda(EDU_{t-1}^{art}+\alpha)} + e^{-\lambda(EDU_{t-1}^{high})} + e^{-\lambda(EDU_{t-1}^{det})}}$$

$$P_t^{det} = \frac{e^{-\lambda(EDU_{t-1}^{det})}}{e^{-\lambda(EDU_{t-1}^{art}+\alpha)} + e^{-\lambda(EDU_{t-1}^{high})} + e^{-\lambda(EDU_{t-1}^{det})}}$$

The next and final step in the prediction process is to calculate the individual's log likelihood that they will choose the chosen routes over the 90-day period, found by summing the natural logs of the likelihood of choosing the chosen routes for each of the 90 days in the prediction period. The general formula for this calculation is shown below:

$$LH_t = \sum_{i=31}^{t} \ln P_i^{chosen}$$

$$LH_t = likelihood\ on\ day\ t; t \geq 31$$

$$P_i^{chosen} = probability\ of\ chosen\ route\ of\ day\ i$$

The total log likelihood is used to compare how well the disaggregate analysis predicts an individual's route choice. The closer the log likelihood is to zero (likelihood closer to 1), then the better the model has predicted an individual's route choice with a given set of risk parameter, arterial bias and scale parameters. The likelihood for an individual is calculated for all possible parameter groupings between the c-value, arterial bias, and logit scale. The c-value in the grouping with the total likelihood that is closest to zero will be used to represent that person in construction of the risk attitude distribution for the entire experiment session.

Calibration Results

This section presents the disaggregate analysis results for each session in the incident case. Tables 1, 2, 3, and 4 below shows the individual risk analysis of every participant in sessions A1, A2, A3, and A4, respectively. For each individual, their predicted c-value, arterial bias, logit scale, and optimal likelihood are presented. Figures 3, 4, 5, and 6 respectively present the risk distributions of A1, A2, A3, and A4, as predicted by the disaggregate analysis.

Table 1: Disaggregate Results for Session A1

User	c	α	λ	Likelihood
A1u1	1.1	100.00	0.01	-71.015
A1u2*	0.5	-1.00	0.10	-97.484
A1u3	0.3	0.10	10.00	-76.009
A1u4	0.5	1.00	1.00	-73.76
A1u5	1.2	100.00	0.10	-88.091
A1u6	1.1	-100.00	0.01	-61.373
A1u7	0.1	100.00	0.01	-94.58
A1u8*	0.1	0.01	10.00	-97.766
A1u9	0.1	-100.00	0.01	-83.63
A1u10	0.2	0.10	10.00	-83.894
A1u11	1.1	100.00	0.10	-38.977
A1u12	1.1	-100.00	0.01	-81.69
A1u13	1.5	-100.00	0.01	-66.419
A1u14	0.1	-100.00	0.10	-40.016
A1u15	0.4	-1.00	1.00	-59.839
A1u16	1.5	-100.00	0.01	-72.741

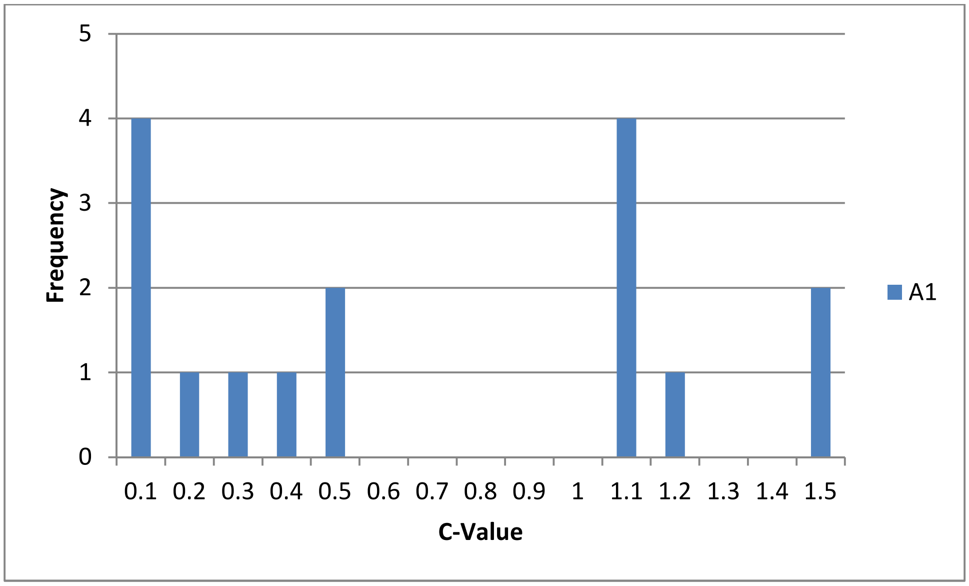


Figure 2: Predicted Risk Parameter Distribution for Session A1

Table 2: Disaggregate Results for Session A2

User	c	α	λ	Likelihood
A2u1	0.1	0.01	100.00	-61.714
A2u2	0.2	-0.01	10.00	-90.85
A2u3	0.1	0.01	100.00	-65.245
A2u4	1.5	Multiple		0
A2u5	0.1	-0.01	100.00	-36.338
A2u6	0.1	0.01	100.00	-86.421
A2u7	1.3	-10.00	0.01	-87.131
A2u8	0.2	-0.01	100.00	-22.23
A2u9	0.1	-0.01	100.00	-49.374
A2u10	0.2	-0.10	10.00	-70.882
A2u11	0.1	0.01	100.00	-63.436
A2u12	0.1	0.00	100.00	-80.765
A2u13	0.2	-0.01	10.00	-95.251
A2u14*	0.2	0.01	10.00	-97.142
A2u15	0.1	0.01	100.00	-53.104
A2u16	0.2	0.10	10.00	-92.195

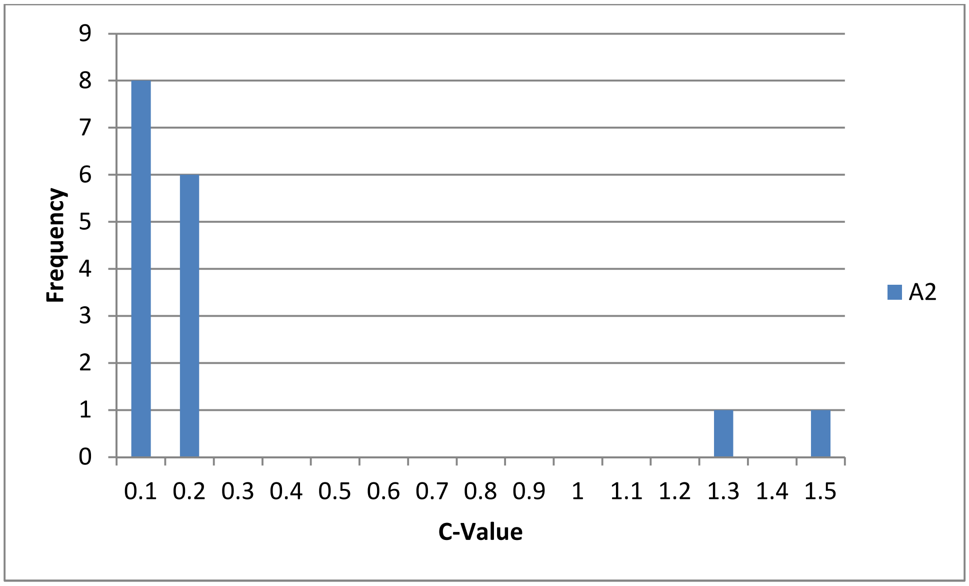


Figure 3: Predicted Risk Parameter Distribution for Session A2

Table 3: Disaggregate Results for Session A3

User	c	α	λ	Likelihood
A3u1*	0.1	-1.00	0.10	-98.377
A3u2	0.2	0.10	100.00	-51.013
A3u3	0.1	0.01	100.00	-83.189
A3u4	0.1	0.01	100.00	-77.056
A3u5	0.1	0.01	100.00	-90.821
A3u6	0.1	-100.00	0.01	-92.631
A3u7*	0.1	0.01	100.00	-98.051
A3u8	0.1	0.00	100.00	-68.283
A3u9	0.1	0.01	100.00	-92.173
A3u10*	0.4	0.01	1.00	-97.323
A3u11*	0.1	0.01	100.00	-97.825
A3u12	0.2	0.10	100.00	-30.365
A3u13	1.1	-100.00	0.01	-82.819
A3u14	0.1	0.00	100.00	-76.574
A3u15*	0.1	0.10	10.00	-97.059
A3u16	0.2	0.10	100.00	-51.013

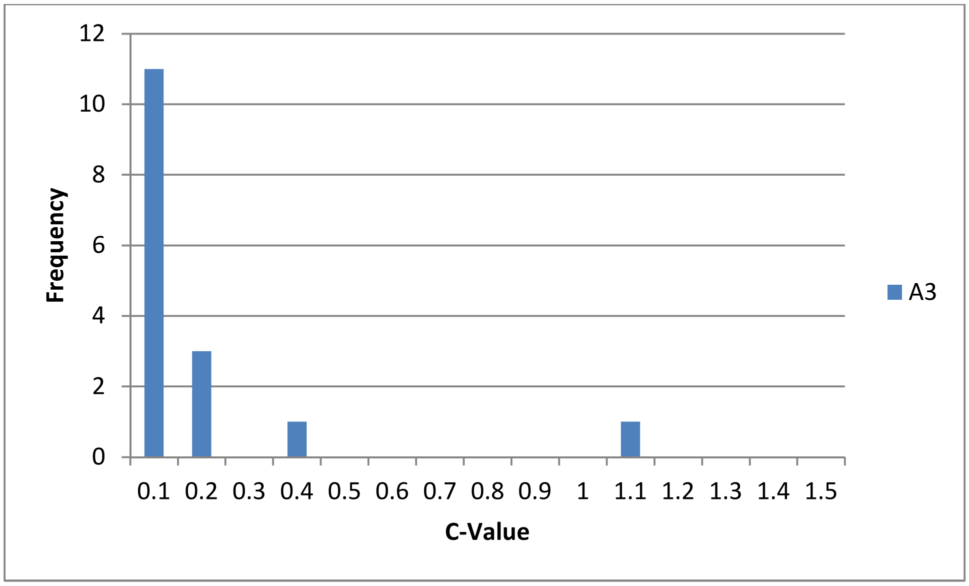


Figure 4: Predicted Risk Parameter Distribution for Session A3

Table 4: Disaggregate Results for Session A4

User	c	α	λ	Likelihood
A4u1	1.1	-100.00	0.01	-83.292
A4u2	0.4	0.10	10.00	-60.820
A4u3	0.1	0.01	100.00	-53.473
A4u4*	0.1	-10.00	0.01	-98.591
A4u5	0.2	-0.01	10.00	-92.416
A4u6	0.2	-0.10	10.00	-78.570
A4u7	0.1	0.01	100.00	-71.072
A4u8	0.3	-0.10	10.00	-70.831
A4u9	0.1	0.00	100.00	-75.368
A4u10	0.2	0.00	10.00	-94.804
A4u11	0.3	-1.00	1.00	-86.208
A4u12	0.4	0.01	10.00	-32.285
A4u13	0.1	-0.10	10.00	-86.167
A4u14	0.1	0.00	100.00	-75.998
A4u15	0.1	0.01	100.00	-55.028
A4u16	0.3	100.00	10.00	-53.484

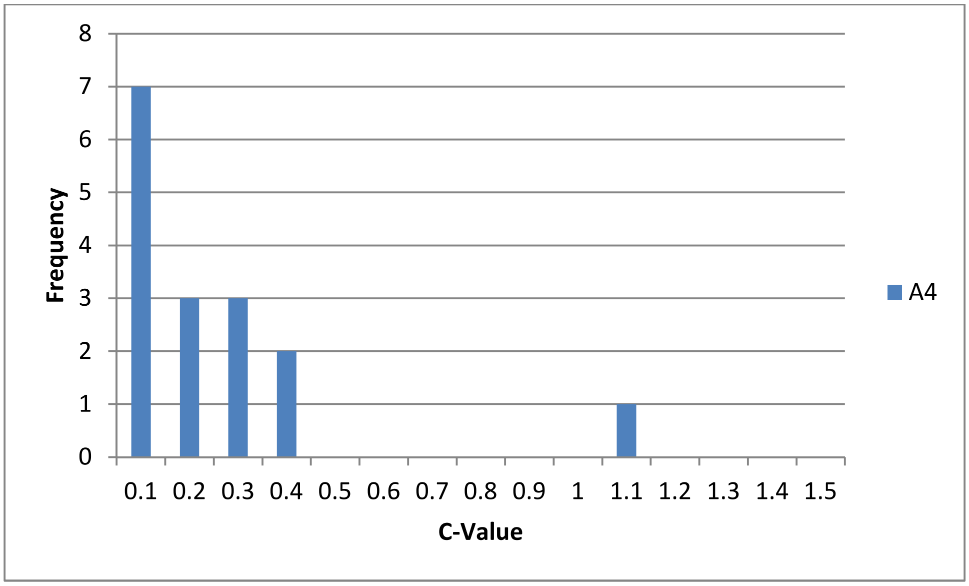


Figure 5: Predicted Risk Parameter Distribution for Session A4

The results suggest that most of the individuals in the session are risk-seeking, but there are certain individuals whose optimal likelihoods seemed questionable, as indicated by the starred user IDs. The reason these likelihoods are questionable is that their values are so close to the log likelihood of a purely random guess (assigning 1/3 choice probability to each alternative), -98.8751, suggesting that the model does not explain the participant's behavior better than a purely random guess.

There are a few reasons as to why the disaggregate calibration may not correctly characterize the individual risk attitudes for all participants, particularly for session A3. The simplest explanation could be that the range of values for the arterial bias and the logit scale is limited. The arterial bias ranges between 100 and -100 in increments of factors of 10, including zero. The logit scale has the same range and increments, excluding zero. The optimal bias and scale values could be within the limits of the range or even outside the assumed range. Experimentation with the range of values could provide better results for individuals.

Other reasons for incorrect predictions could be in part of lack of knowledge about individual learning behavior. The user equilibrium model does not incorporate how individuals learn about the network and perceive the travel times after each day's route choice.

For example, there is a lack of any recency effect in the model. The recency effect is the assumption that individuals will only remember the travel time of only a couple of recent days. With the current equilibrium model, it is assumed that individuals will remember the travel times they experienced over all previous days. Another concept related to the recency effect suggests that individuals will remember days where the travel time was small more than when it was large as the probability of incident is less than 0.5. Incorporation of this effect may explain why individuals seem to be risk-seeking in all predictions. Another possible explanation for such risk-seeking behavior is because individuals were told that there was a 0.25 disruption probability on the highway and want to try to predict when the highway is not experiencing an incident, so they choose the highway most frequently.

While the disaggregate analysis results still needs improvement with respect to parameter sets, the results do show that each session has a variety of c-values among participants. These results do in fact show that it is safe to assume that the risk parameter distribution for a network includes both risk-seeking and risk-averse groups and the distribution is not uniform.

3.3.2 Aggregate Analysis

The aggregate calibration method creates multiple risk parameter distributions to predict the distribution with results that best match the observed average path flows in the experiment sessions. The distribution is first split between two groups: risk-averse (c=0.1-1.0) and risk-seeking (c=1.0-1.5), based on the disaggregate analysis results. The range of the c-values was arbitrarily chosen for this analysis. Previous analysis of an overall uniform distribution across one range of c-values produced multiple optimal results for risk parameter ranges. The disaggregate analysis later disproved the notion that an overall uniform distribution is best for modeling risk behavior.

Next, a total number of risk groups will be assigned to the risk-averse and risk-seeking sides of the distribution. The total number of risk groups allowed for any analysis can range from 2 (e.g. 1 risk-seeking, 1 risk-averse) to 16 (e.g. 1 risk-seeking, 15 risk-averse). An upper limit of 16 was chosen to reflect each participant having a different c-value assignment. With this set up, three distribution scenarios are analyzed: 1) Risk-seeking individuals have one risk group while risk-averse individuals have 2-15 risk groups, 2) Risk-seeking and risk-averse individuals can have anywhere between 2-14 risk groups while maintaining the total number of risk groups no greater than 16, and 3) Risk-seeking individuals have 2-15 possible risk groups while risk-averse individuals have only one.

Based on the range and number of risk groups for the risk-seeking and risk-averse sides of the distribution, each has various parameters that must be set before allocating flows to each c-value in the distribution. To specify the range of c-values, it is important to know the minimum and maximum c-values for each side of the distribution. If a side has only one risk group, then this will be known as the minimum c-value for that side and no maximum value will be used in the analysis.

When there is a range of c-values that will be analyzed in the model, then more parameters are required to determine the distribution of c-values within that range. For any range, it is assumed the number of risk groups will be uniformly distributed between the minimum and maximum values. Once a range has been established, the average c-value (μ) of the range will be calculated. Once the average is found, the difference (σ) between the average and the lower bound is then calculated. This is done to determine the c-value for the other risk groups in the range, using the following formulas:

$$\mu = \frac{c_{high} + c_{low}}{2}$$

$$\sigma = \mu - c_{low}$$

$$c_i = c_{i-1} + \frac{2\sigma}{r-1}; i > 1; c_1 = c_{low}$$

$$c_i = c - value\ for\ risk\ group\ i$$

$$r = total\ number\ of\ risk\ groups\ within\ risk\ attitude$$

Once the range of c-values for each side of the distribution is determined, flows can then be allocated to each c-value within this distribution. First, the total population is divided into two groups: risk-seeking and risk-averse. This split in population presents two more parameters (total risk-seeking and total risk-averse) that must be calibrated with the model. Once the risk-seeking and risk-averse populations are decided, each population is evenly divided into various risk groups. How much of a population a risk group receives depends on the number of risk groups within that population. If the population is less than the number of risk groups, then the distribution is excluded from the analysis. This is meant to represent the condition that at least one individual is represented within a c-value. After the flows are allocated to each risk population and c-value user class, the solution algorithm can be run.

A cutoff limit of 0.0001 is used as the convergence criterion of the MSA. Therefore, if the difference of two iterations' expected disutilities for each user class was less than 0.0001, then equilibrium has been met, and the accuracy of the predicted path flows for each route could be analyzed.

In order to test the accuracy of the model's prediction, the predicted path flows were compared to the observed average path flows for an experiment session. The average path flows for the experiment's last 90 days was used to test the model's accuracy. Using the last 90 days for path flow average was done in order to better represent more stable route choices from the experiment participants. At the beginning of the experiment, participants would like to explore the various routes in the network before making more habitual and consistent route choices that would better reflect their risk attitude. This exploratory behavior was typically seen within the first 30 days of the experiment.

To calculate the fitness of the model's predicted path flows, the normalized mean square error, or RMSN, was used. The RMSN for the model is calculated by using the following equation:

$$RMSN = \frac{p * \sqrt{\frac{\left(q_{pred}^{art} - q_{obs}^{art}\right)^2 + \left(q_{pred}^{high} - q_{obs}^{high}\right)^2 + \left(q_{pred}^{det} - q_{obs}^{det}\right)^2}{p}}}{Q}$$

$q_{pred}^{art} = predicted\ arterial\ flow\ (veh)$

$q_{obs}^{art} = observed\ arterial\ flow\ (veh)$

$q_{pred}^{high} = predicted\ highway\ flow\ (veh)$

$q_{obs}^{high} = observed\ highway\ flow\ (veh)$

$q_{pred}^{det} = predicted\ detour\ flow\ (veh)$

$q_{obs}^{det} = observed\ detour\ flow\ (veh)$

$p = total\ number\ of\ route\ choices$

$Q = total\ demand\ (veh)$

The model was then designed to display all possible distributions that, when analyzed, would produce an RMSN less than 0.10. While having an RMSN of 0.0 would be ideal, as it signifies perfectly matching route choice predictions across all paths, any model predictions with an RMSN less than 0.10 were considered acceptable.

Calibration Results

Table 5: Aggregate Risk Attitude Distribution Prediction Results

Session	Predicted Distribution							
	Flow Total (0.1-1.0)	Risk Groups	c_{low}	c_{high}	Flow Total (1.0-1.5)	Risk Groups	c_{low}	c_{high}
A1	6	1	0.1	n/a	10	10	1.3	1.4
A2	12	1	0.3	n/a	4	4	1.0	1.1
A3	8	1	0.1	n/a	8	2	1.0	1.1
A4	8	1	0.3	n/a	8	8	1.2	1.3

Table 6: Aggregate Path Flow Predictions and RMSN Results

Session	Observed Flows			Predicted Flows			RMSN
	Arterial	Highway	Detour	Arterial	Highway	Detour	
A1	8.00	6.22	1.79	7.79	6.00	2.21	0.0561
A2	7.47	6.68	1.86	7.55	6.40	2.05	0.0380
A3	6.62	7.69	1.69	6.84	7.56	1.60	0.0841
A4	7.22	6.44	2.33	7.55	6.40	2.06	0.0465

Table 5 shows the best risk parameter distributions for the four experiment sessions during the incident case. Predicted path flows based on the risk parameter distributions for each session is shown above in Table 6 along with the RMSN value for the predicted path flows. Session A2 has the best path flow prediction of all the incident case sessions, having an RMSN of 0.038. A2 also has the highest number of risk seeking individuals as it has 12 individuals with a c-value of 0.3. It is the most risk-seeking of the incident sessions. In contrast, the most "risk-averse" of the incident sessions is A1, with 10 individuals with a c-value between 1.3 and 1.4.

Collectively, all four sessions do show a general trend with representing the risk distribution. In each session, there is only one risk-seeking group (with the level of risk seeking varying across sessions and multiple users), while multiple risk-averse groups exist.

This analysis assumes uniform distribution of flows between risk groups in either the risk-seeking or risk-averse domain; it would be of great interest to see if distributing the flow between risk groups differently can improve the prediction results of the model.

CHAPTER 4

MODELING USER EQUILIBRIUM WITH ADAPTIVE ROUTE CHOICE UNDER REAL-TIME INFORMATION

The following section details the development of a model to predict user equilibrium traffic flows with the inclusion of real-time information. This is done by incorporating a user class who has information access to traffic conditions within the modeled network. This chapter will discuss the how equilibrium conditions are met with real-time information, the changes made to the solution algorithm and applications of the model. The model has been designed for calibration with data from the information case experiment sessions, even though the calibration has not been carried out.

4.1 User Equilibrium Conditions with Real-Time Information

The conventional user equilibrium condition in a static and deterministic network is generalized to the stochastic network with heterogeneous information access (and homogeneous risk neutrality) in this thesis as follows: at user equilibrium, all used routing policies have the same and minimum mean travel time for each origin-destination pair and information access class.

There will be two possible user classes used to satisfy equilibrium conditions, users with information and users without information. This depends on the parameter known as the market penetration rate, or MPR. The MPR proportionally divides the total population into the two user classes. When MPR=0, then no one has information and simulations based on the incident case can be performed. When MPR=1, then everyone has information, recreating conditions for the information case of the experiment. The previous two conditions only deal with one user class in the equilibrium solution, but both user classes will be a part of the equilibrium solution if the MPR is between 0 and 1. Even though the potential number of user classes could be larger with the inclusion of heterogeneous risk attitude, for simplicity it is assumed that all individuals are risk neutral (c=1.0) when making route choices in the model.

The type of user class can also have an effect on the number of choice alternatives available for travelers. If there is no information available to travelers, then there are three possible route choices for the experiment: the arterial, the detour, or the highway. If travelers do have information on the highway's traffic conditions, then there are five possible route choices: the arterial, the detour, the highway, and two non-trivial routing policies. A routing policy is a routing decision rule that maps from each decision node to the next link depending on available traffic information on that node. A routing policy can be manifested as multiple paths over different realizations of the information outcomes. A routing policy is a generalization of a path, and a path is a specialization of a routing policy. The first three routing policies are simply the three paths. A traveler following routing policy 4 will take the connector to the information node, and then if there is an incident on the highway, take the detour. If there is no incident on the highway, then he will take the highway. This is the most intuitive routing policy to adopt, as a traveler might try to avoid any road incidents or construction. Routing policy 5, on the other hand, is the exact opposite of routing policy 4. A traveler following routing policy 5 will still take the connector; however, if there is an incident, he will take the highway. Otherwise, the traveler will take the detour. A traveler adopting this routing policy may think that others will have avoided the incident, so by taking the highway with an incident, his travel time may be smaller.

4.2 Solution Algorithm

The MSA will be used as the solution algorithm like the heterogeneous risk attitude solution algorithm. The main operation is still to move flows to alternatives with minimum expected utilities, but the alternatives might be routing policies for the user class with information access.

4.3 Applications

Although the model has not yet been calibrated using data from the information case experiment sessions, its application can be demonstrated using a numerical example. With this model, sensitivity analyses can be performed to see how well a road network will perform using various disruption probabilities and MPR. A sample sensitivity analysis was used with the network design in Figure 7, shown below.

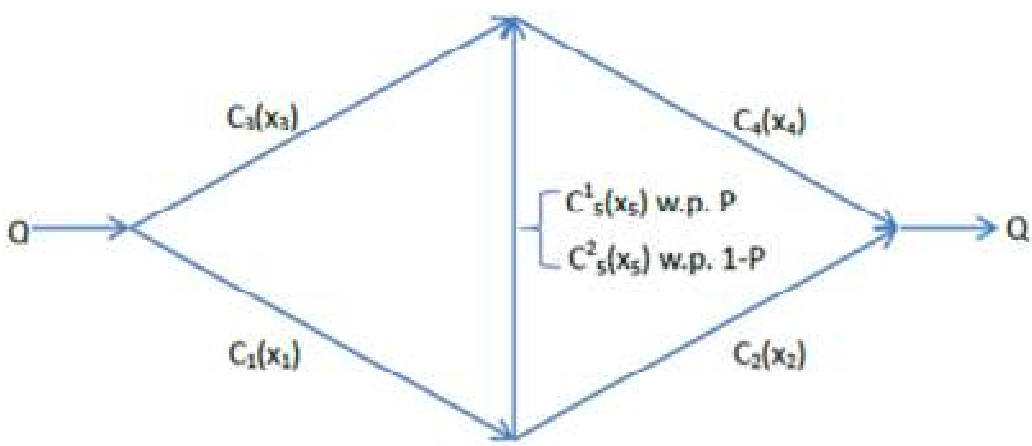


Figure 6: Sample Network for Sensitivity Analysis

Links 1, 2, 3, and 4 will have increasing, deterministic link performance functions while link 5 will be stochastic, where an incident with a given probability P will greatly increase the link's congestion rate, resulting in a higher travel time. For example, C^1_5 will be much larger than C^2_5 for a given link flow x_5. This is supposed to represent the reduced capacity on the link.

The stochasticity of link 5's performance function will have an effect on the network performance. The disruption probability will determine the routing policy choices for those with information, which will have various effects on the network. On one hand, an increased chance of disruption could imply a higher travel time on link 5 with the same flow. On the other hand, flows may be re-distributed because of the change in disruption probability, and might partially relieve the adverse effect of increased disruption probability but at the same time make other routes more crowded. In a particular case, reducing the disruption probability on the incident-prone link could increase the total travel time and recreate the Braess Paradox. It is thus important to use a model to analyze these effects in a case by case basis.

With regards to real-time information, having some with information could help improve the network's performance; however, too high of an MPR could decrease the network's overall performance. Controlling the amount of information available could be helpful, as providing too much information may be counter-productive in improving traffic conditions. Like analyzing disruption effects, analyzing MPR effects should be done in a case by case basis.

For the sample sensitivity analysis, the network has the following link performance functions:

$C_1(x_1)= C_4(x_4)=4.5x$

$C_2(x_2)= C_3(x_3)=50+3x$

$C_5(x_5)=3+0.5x$ w.p. $(1-P)$; $3+45x$ w.p. P

P=Disruption Probability

Initial analysis of the created network illustrates some observed trends with respect to disruption probability and information access. The following two figures show the total travel time of the network with respect to increasing disruption probability and MPR, respectively.

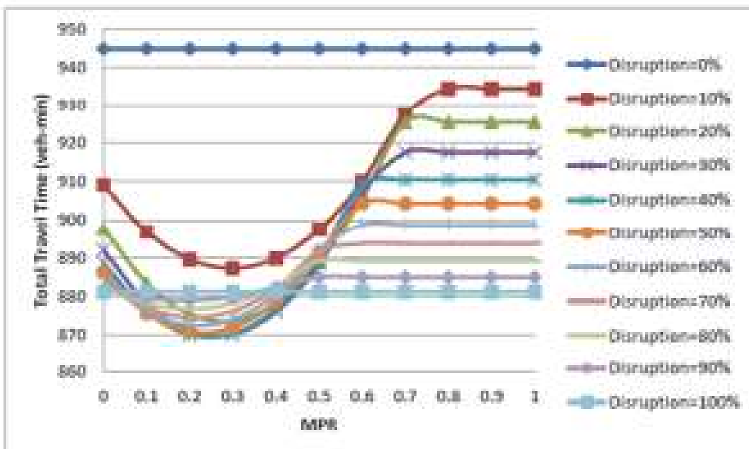


Figure 8: Total Travel Time Analysis under Various Disruption Probabilities

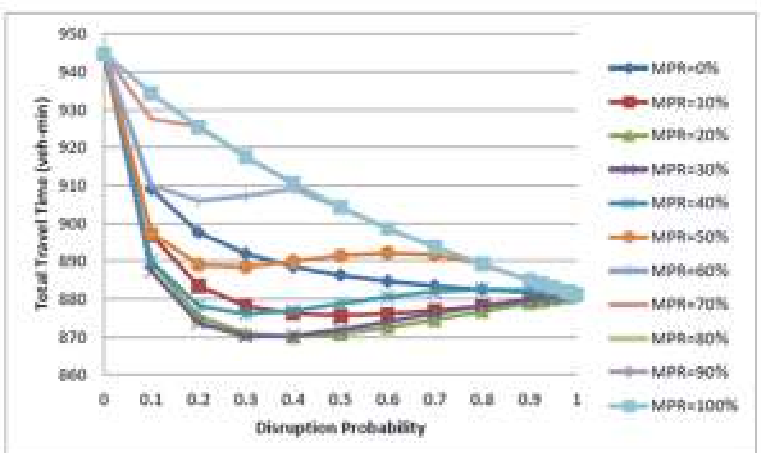


Figure 7: Total Travel Time Analysis under Various MPR

Figure 8 shows that as the MPR increases, the total travel time of the network will generally decrease when MPR is 10-30%, but then it will start to increase once the MPR is 40%. This affirms the notion that too much information is not necessarily an improvement for the system. Figure 9 shows that as the MPR increases, regardless of the disruption probability, the total travel time does not change drastically when the MPR is greater than 70%.

With respect to disruption probability, Figure 9 illustrates that depending on the MPR, the total travel time will change differently as the disruption probability increases. Initially, when MPR is 10-30%, the total travel time will decrease and then slightly increase when the disruption probability reaches around 40%. Then, when MPR is 40-60%, the total travel time will decrease sharply, then slightly increase before decreasing in order to converge with the MPR=100% trend line. Finally, the total travel time will decrease as the disruption probability increases when the MPR is 70-100%.

CHAPTER 5

CONCLUSIONS AND FUTURE WORK

The following presents the overall concluding remarks on the results of the ongoing development of two user equilibrium models, one with respect to heterogeneous risk attitude and the other to real-time information access. Both models achieve user equilibrium by minimizing the expected disutility for each route choice across all user classes. The equilibrium conditions were heuristically solved using MSA, as used in previous studies. The heterogeneous risk attitude model, however, was calibrated using route choice data from the experiment in Lu et al. (2012). The real-time information model has not yet been calibrated but has been used to perform a sensitivity analysis to demonstrate its possible applications.

Initial calibrations of the heterogeneous risk attitude show promising results in predicting traveler behavior and user equilibrium conditions, although more investigation is needed to see if the initial risk distribution predictions are accurate. The disaggregate analysis predicts that an overwhelming majority of users in the experiment are risk-seeking, contrary to the results of the previous risk attitude studies covered in the literature review. This is particularly interesting as the previous studies look at the choices of each individual to make a generalized assumption of an entire study, much like the purpose of the disaggregate analysis. While the disaggregate analysis predicts fairly risk-seeking behavior among all participants, the aggregate analysis predicts more risk-averse behavior among the participants than the disaggregate analysis, which is supportive of the previous risk attitude studies. The only exception to this observation is session A2, where there was a more risk-seeking prediction among participants.

With regards to future work, the disaggregate analysis will need the most improvements, as evidenced by the very small likelihoods generated in Table 3 for participants in experiment session A3. As this analysis method looks at the route choices of each individual, it may be important to add parameters that reflect travelers' memory of network travel times or learning effects, as seen in Lu et al. (2012). The aggregate analysis is already showing encouraging results with fairly small RMSN values across all four incident case experiments. It would be important to test out different risk parameter distributions in future research to see if different distribution patterns will produce better fit in the solution algorithm.

The real-time information model shows potential in predicting network performance based on various disruption probability and MPR combinations. The preliminary results of the model do support the assumption that providing more information about the network does not necessarily improve the network's performance. With respect to varying disruption probabilities, model results show that having some disruption in the network will actually improve network conditions. A possible suggestion for this finding could be that the disruption will encourage travelers to make more proactive route choices. It would be best to adapt the model to the network and experiment conditions in Lu et al. (2012) and calibrate the model with route choice data from the information sessions. Parameters to be calibrated would be associated with traveler characteristics inherent in other route choice models incorporating real-time information.

It would be ideal for future work to consolidate both into a single user equilibrium model. This model would be able to predict user equilibrium conditions for any given road network based on the population's risk attitude distribution, multiple disruption probabilities across various links, and various MPRs. Research in the development of a consolidated model could help make accurate traffic predictions for a city based on both external (traffic disruptions) and internal (traveler risk attitude) forces. Incorporating both may help reduce delay for travelers everywhere.

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